Girraween High School

2017

TRIAL HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time: 5 minutes
- Working time: 2 Hours
- Write using a black or blue pen
- Board approved calculators may be used
- Laminated reference sheets are provided
- Answer multiple choice questions by completely colouring in the appropriate circle on your multiple choice answer sheet on the front page of your answer booklet.
- In questions 11-15 start all questions on a separate page in your answer booklet and show all relevant mathematical reasoning and/or calculations.

Total Marks: 85

Section 1 **10 Marks**

- Attempt Q1 - Q10
- Allow about 15 minutes for this section

Section 2 **75 marks**

- Attempt Q11 - Q15
- Allow about 1 hour and 45 minutes for this section

MATHEMATICS

Trial Examination

For questions 1-10, fill in the response oval corresponding to the correct answer on your Multiple choice answer sheet.

1. What is the value of $\lim_{x\to\infty} \frac{\sin\left(\frac{1}{2}\right)x}{2x}$?

A) 0 B) $\frac{1}{4}$ C) 1 D) 4

2. Which of the following is a simplification of $\cot 2x + \tan x$?

A) $\sec 2x$ B) $\sec x$ C) $\cos ecx$ D) $\cos ec2x$

3. The equation $x^3 + bx^2 + cx + d = 0$ has roots $\alpha, \beta\ \gamma$. What is the value of $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha}$?

A) $-b$ B) $\frac{-b}{d}$ C) $\frac{b}{d}$ D) b

4. Which of the following is a simplification of $4\log_e \sqrt{e^x}$?

A) $4\sqrt{x}$ B) $\frac{1}{2}x$ C) $2x$ D) x^2

5. Which of the following is an expression for $\int \sin^2 6x\, dx$?

A) $\frac{x}{2} - \frac{1}{12}\sin 6x + c$ B) $\frac{x}{2} + \frac{1}{12}\sin 6x + c$

C) $\frac{x}{2} - \frac{1}{24}\sin 12x + c$ D) $\frac{x}{2} + \frac{1}{24}\sin 12x + c$

6. Four female and four male students are to be seated around a circular table.

How many ways can this be done if the males and females must alternate?

A) $4!\times 4!$ B) $3!\times 4!$ C) $3!\times 3!$ D) $2\times 3!\times 4!$

7. The acute angle between the lines $2x - y = 0$ and $kx - y = 0$ is equal to $\frac{\pi}{4}$.

What is the value of k?

A) $k = 3$ or $k = -\frac{1}{3}$ B) $k = -3$ or $k = \frac{1}{3}$

C) $k = -3$ or $k = -\frac{1}{3}$ D) $k = 3$ or $k = \frac{1}{3}$

8. Which of the following is equivalent to $\int \frac{dx}{4x^2+9}$, ignoring the constant of integration?

A) $\tan^{-1}\frac{2x}{3}$ B) $\frac{1}{6}\tan^{-1}\frac{2x}{3}$ C) $\frac{2}{3}\tan^{-1}\frac{2x}{3}$ D) $\frac{3}{2}\tan^{-1}\frac{2x}{3}$

9. What is the term independent of x in the expansion of $\left(x^3 + \frac{2}{x}\right)^{20}$?

A) $\binom{20}{10}2^{20}$ B) $\binom{20}{5}2^{15}$ C) $\binom{20}{4}2^{16}$ D) $\binom{20}{5}2^{25}$

10. Which of the following is an expression for $\frac{d}{dx}\sin^{-1}(2x-1)$?

A) $\frac{-1}{\sqrt{x(x-1)}}$ B) $\frac{-1}{2\sqrt{x(x-1)}}$ C) $\frac{1}{2\sqrt{x(1-x)}}$ D) $\frac{1}{\sqrt{x(1-x)}}$

Question11.(15 marks)- (show all necessary working) **marks**

a) $A(-3,1)$ and $B(1,-2)$ are two points. Find the coordinates of the point P that divides the interval AB externally in the ratio 3:1. **2**

b) Find $\int \frac{1+2x}{1+x^2}dx.$ **2**

c) Use the substitution $x=u-2$ to evaluate $\int_{-1}^{2} \frac{3x+5}{\sqrt{x+2}}dx.$ **3**

d) Use mathematical induction to prove that $3^{2n+4}-2^{2n}$ is divisible by 5, for $n \geq 1$. **4**

(e) i) Show that $\frac{\sin 2x}{1+\cos 2x} = \tan x$ **2**

ii) Hence show that $\tan 15^0 + \cot 15^0 = 4$ **2**

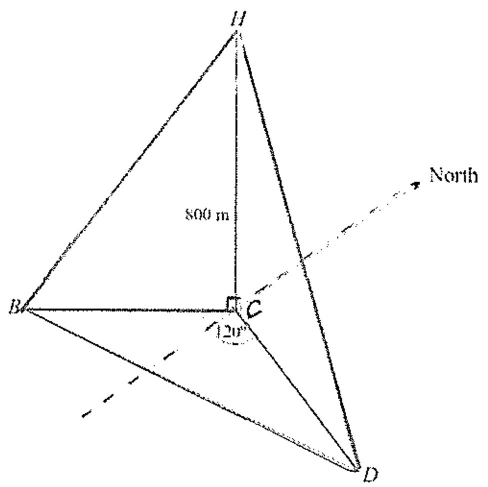
Question 12.(15 marks)

a)i) Find $\frac{d}{dx}\left(\tan^{-1}\frac{x}{3}\right)^2$ **2**

ii) Hence find the exact value of $\int_0^{\sqrt{3}} \frac{\tan^{-1}\frac{x}{3}}{x^2+9}dx$ **2**

b) The region enclosed by the curve $y=\sin^{-1}x$ and the $y-$axis between $y=0$ and $y=\frac{\pi}{3}$ is rotated about the $y-$axis to form a solid. Find the exact volume of the solid of revolution formed. **3**

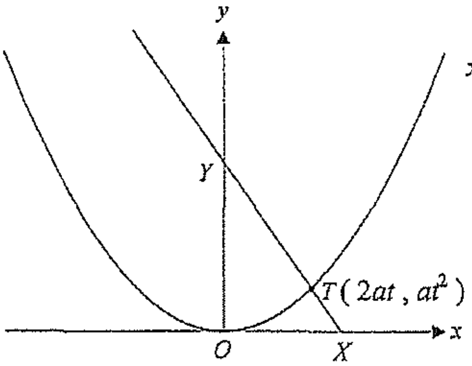
c)



The diagram above shows a hot air balloon at point H with altitude $800m$.
The passengers in the balloon can see a barn and a dam below, at points B and D respectively. Point C is directly below the hot airballoon. From the hot air balloon's position, the barn has a bearing of 250^0 and the dam has a bearing of 130^0, and $\angle BCD = 120^0$. The angles of depression to the barn and the dam are 50^0 and 30^0 respectively.

How far is the barn from the dam, to the nearest metre? 4

d) In the diagram, $T(2at, at^2)$ is a point on the parabola $x^2 = 4ay$.



i) Show that the normal to the parabola at T has equation $x + ty = 2at + at^3$. **2**

ii) This normal cuts the x and y axes at X and Y respectively.

Show that $\dfrac{TX}{TY} = \dfrac{t^2}{2}$ **2**

Question 13.(15 marks)

a) A particle is performing Simple Harmonic Motion in a straight line. At time t seconds it has displacement x metres from a fixed point O on the line given by $x = 6\cos^2 t - 2$.

i) Show that $\ddot{x} = -4(x-1)$. **2**

ii) Find the centre and period of the motion. **2**

b) A particle is moving in a straight line. At time t seconds it has displacement x metres from a fixed point O on the line. Its velocity v m/s is given by $v = -\frac{1}{8}x^3$. The particle is initially 2 metres to the right of O.

i) Show that the acceleration a, is given by : $a = \frac{3}{64}x^5$. **2**

ii) Find an expression for x in terms of t. **3**

c) Consider the function $f(x) = (x+2)^2 - 9$, $-2 \le x \le 2$.

i) Find the equation of the inverse function $f^{-1}(x)$. **1**

ii) On the same diagram, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$, showing clearly the coordinates of the end points and the intercepts on the coordinate axes. **3**

iii) Find the x-coordinate of the point of intersection of the curves $y = f(x)$ and $y = f^{-1}(x)$, giving the answer in simplest exact form. **2**

Question 14(15 marks).

a) The coefficients of x^2 and x^{-1} in the expansion of $\left(ax-\frac{b}{x^2}\right)^5$ are the same.

Show that $a+2b=0$, where a and b are positive integers. **3**

b) Show that $\tan^{-1}\left(\frac{3}{4}\right)+\cos^{-1}\left(\frac{3}{5}\right)=\frac{\pi}{2}$ **2**

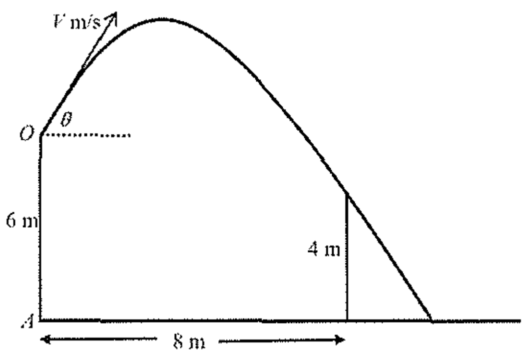
c) i) Neatly sketch the graph of $y=\sin^{-1}\left(\frac{x}{2}\right)$ clearly indicating the domain and range. **2**

ii) By considering the graph in part(i), find the exact value of:

$$\int_0^1 \sin^{-1}\left(\frac{x}{2}\right)dx$$ **2**

d) A projectile is fired from a point O, which is 6 metres above horizontal ground, with initial velocity Vm/s at an angle of θ to the horizontal.

There is a thin vertical post which is 4 metres high and 8 metres horizontally away from a point A, directly below O, as shown in the diagram below.



The equations of motion are given by:

$x=Vt\cos\theta$ and $y=Vt\sin\theta-4.9t^2$ (Do Not prove this)

i) If 2 seconds after projection, the projectile passes just above the top of the post, show that $\tan\theta=2.2$ **2**

ii) Show that the projectile hits the ground approximately 0.3 seconds after passing over the post. **2**

iii) Find the angle that the projectile makes with the ground when it hits the ground, correct to the nearest degree. **2**

Question 15.(15 marks)

a) $P(x) = ax^3 - 7x^2 + kx + 4$ has $x-4$ as a factor. When $P(x)$ is divided by $(x-1)$, the remainder is -6.

i) Determine the values of a and k. **2**

ii) Evaluate $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$. **1**

b) Consider the series $\log_e \frac{a^3}{\sqrt{b}} + \log_e \frac{a^3}{b} + \log_e \frac{a^3}{b\sqrt{b}} + \log_e \frac{a^3}{b^2} + \ldots\ldots$

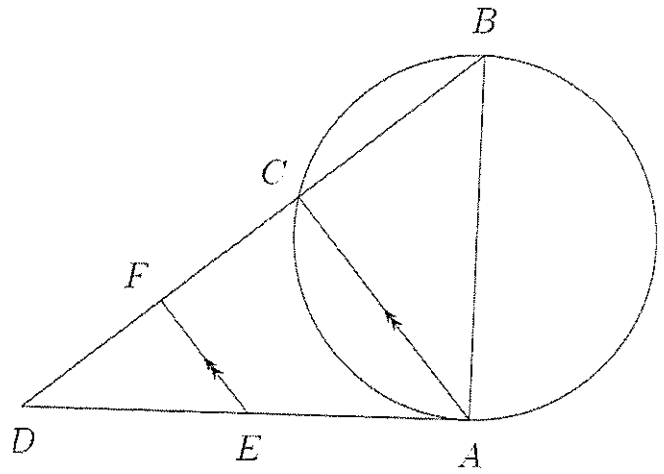
i) Prove that the series is an arithmetic series and state the common difference. **2**

ii) Find an expression for the sum of the first 23 terms of the series, giving your answer in the form $\log_e \frac{a^m}{b^n}$ where m and n are integers. **2**

c) Use the substitution $u = e^{4x} + 9$ to give the exact value of :

$$\int_0^{\ln 2} \frac{3e^{4x}}{\sqrt{e^{4x}+9}}\,dx$$ **2**

d) AB is a diameter of the circle and C is a point on the circle. The tangent to the circle at A meets BC produced at D. E is apoint on AD and F is a point on CD such that $EF \parallel AC$



i) Copy the diagram in your answer booklet and state why $\angle EAC = \angle ABC$ **1**

ii) Hence show that $EABF$ is a cyclic quadrilateral. **2**

iii) Show that BE is a diameter of the cicle through E, A, B and F. **1**

e) Four adults and four children are to be seated around a circular table.
A particular child cannot sit next to any adult and a particular adult cannot sit next to any child.
Find how many such arrangements are possible. **2**

End of examination!!!

Girraween 2017 Ext 1 Trial Solutions

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
B	D	C	C	C	D	B	B	B	D

Notes on Multiple Choice:

Q1: Divide top and bottom through by 4.

Q2:

$$\frac{\sin x}{\cos x}+\frac{\left(\cos^2 x-\sin^2 x\right)}{2\sin x\cos x}=\frac{2\sin^2 x+\left(\cos^2 x-\sin^2 x\right)}{2\sin x\cos x}=\frac{1}{2\sin x\cos x}=\frac{1}{\sin 2x}=\csc 2x$$

Q3:

$$\frac{\gamma+\alpha+\beta}{\alpha\beta\gamma}=\frac{\left(-\frac{b}{a}\right)}{\left(-\frac{d}{a}\right)}=\frac{b}{d}$$

Q4: $4\log_e e^{\frac{x}{2}}=4\cdot\frac{1}{2}\log_e e^x=2x$

Q5: Recall that $\sin^2 6x=\frac{1}{2}(1-\cos 12x)$

$$\int \sin^2 6x\ dx=\int\frac{1}{2}(1-\cos 12x)\ dx=\frac{1}{2}x-\frac{1}{24}\sin 12x+C$$

Q6:

Doesn't matter where first person sits. Sit him/her down and call that position 1 on the table. Fill the remaining odd positions (clockwise) with the same gender. This accounts for the 3!.

Fill the remaining even positions with the remaining people (4!).

Multiple by 2, as the initial person could be either a male/female $= 2\times 4!\times 3!$

Q7: Gradient of line $2x-y=0$, is 2.

$\frac{2-m}{1+2m}=\pm 1,$

$2-m=1+2m$ or $2-m=-1-2m$

$m=\{\frac{1}{3},-3\}$

Q8: Modify integral to standard form

$$\frac{1}{4}\int\frac{dx}{x^2+\left(\frac{9}{4}\right)}=\frac{1}{6}\tan^{-1}\frac{2x}{3}$$

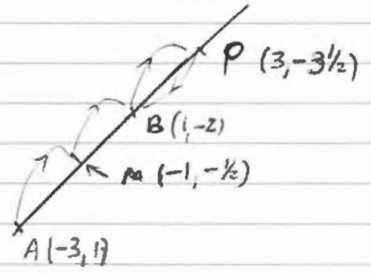
Q9: $\binom{20}{n}2^{20-n}x^{4n-20}$ is the value of each term. Solve the coefficient equal to zero $\Rightarrow n=5$.

Q10: Differentiate

$$\frac{d}{dx}\sin^{-1}(2x-1)=\frac{2}{\sqrt{1-4x^2+4x-1}}=\frac{1}{\sqrt{x(1-x)}}$$

Q11

(a)



Check using formula $P(x, y) = \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n} \right)$

/2

(b) $\int \frac{1+2x}{1+x^2}\,dx = \int \frac{1}{1+x^2} + \frac{2x}{1+x^2}\,dx$

$= \tan^{-1}(x) + \log_e(1+x^2) + C$ /2

(c) Let $x = u - 2$. Limits become $[1, 4]$

that is $u = x + 2$ $\frac{du}{dx} = 1$,

So $I = \int_1^4 \frac{3(u-2)+5}{\sqrt{u}}\,du = \int_1^4 \frac{3u-1}{\sqrt{u}}\,du$

$= \int_1^4 3u^{1/2} - u^{-1/2}\,du$

$= \left[2u^{3/2} - 2u^{1/2}\right]_1^4$

$= 2(8-2) - (2-2)$

$= 12$ /3

11(d) For $n=1$

$3^6 - 2^2$

$= 729 - 4$

$= 725$

which is divisible by 5.

Assume $3^{2k+4} - 2^{2k}$ to be divisible by 5.

Now, for $n = k+1$,

$3^{2(k+1)+4} - 2^{2(k+1)}$

$= 3^{2k+6} - 2^{2k+2}$

$= 3^2(3^{2k+4} - 2^{2k}) + 3^2 \cdot 2^{2k} - 2^2 \cdot 2^{2k}$

$= 9(5K) + (9-4)2^{2k}$ $K \in \mathbb{N}$

$= 5(9K + 2^{2k})$

$= 5\hat{K}$ as required. $\hat{K} \in \mathbb{N}$.

⇒ True for the case when $n=1$, and two consecutive cases

Hence by Principal of Mathematical Induction, the statement is true, ie.

$3^{2n+4} - 2^{2n}$ is divisible by 5 for all $n \in \mathbb{N}$

/4

(e) (i) $\frac{2\sin x \cos x}{1 + (2\cos^2 x - 1)} = \frac{\sin x}{\cos x} = \tan x$ /2

(ii) Using (i), let $x = 15°$ (or $\pi/12$)

$\tan 15° = \frac{1}{2} \cdot \frac{1}{\left(1 + \frac{\sqrt{3}}{2}\right)} = \frac{1}{2+\sqrt{3}}$ & $\cot 15° = \frac{2+\sqrt{3}}{1}$

$= 2 - \sqrt{3}$ * $= 2 + \sqrt{3}$

$\therefore \cot 15 + \tan 15 = 4$. /2

* since $(2-\sqrt{3})(2+\sqrt{3}) = 1$.

End of Q11.

Q12

(a) (i) $\dfrac{6\tan^{-1}(\frac{x}{3})}{x^2+9}$ /2

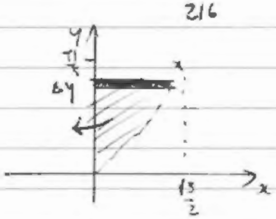
(ii) $I = \dfrac{1}{6}\displaystyle\int_0^{\sqrt{3}} \dfrac{6\tan^{-1}(\frac{x}{3})}{x^2+9}\,dx$

$= \dfrac{1}{6}\displaystyle\int_0^{\sqrt{3}} \dfrac{d}{dx}\left[\tan^{-1}(\tfrac{x}{3})\right]^2 dx$

$= \dfrac{1}{6}\left[\tan^{-1}(\tfrac{x}{3})\right]^2 \Big|_0^{\sqrt{3}}$

$= \dfrac{\pi^2}{216}$ /2

(b)



Typical disk has volume.
$\Delta V = \pi x^2 \Delta y$

$x = \sin y$ (as $y = \sin^{-1}x$)

So $\Delta V = \pi \sin^2 y\, \Delta y$

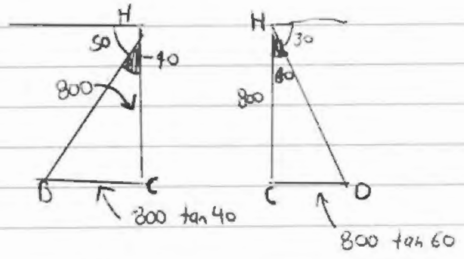
So $V = \pi\displaystyle\int_0^{\frac{\pi}{3}} \sin^2 y\, dy = \pi\int_0^{\frac{\pi}{3}} \left(\tfrac{1}{2} - \tfrac{1}{2}\cos 2y\right) dy$

$= \dfrac{\pi}{2}x - \dfrac{1}{4}\sin 2y \Big|_0^{\frac{\pi}{3}}$

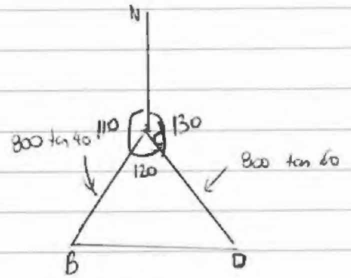
$= \dfrac{\pi^2}{6} - \dfrac{1}{4}\cdot\dfrac{\sqrt{3}}{2}$

$= \dfrac{\pi^2}{6} - \dfrac{\sqrt{3}}{8}$ /3

12 (c)



$\tan\alpha = \frac{O}{A}$
$O = A\tan\alpha$.



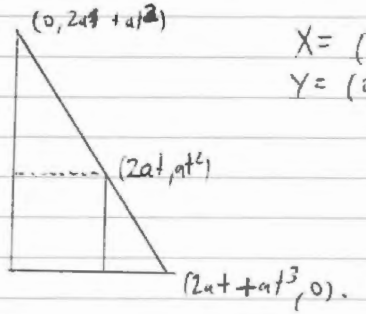
Using cosine rule
$BD^2 = a^2 + b^2 - 2ab\cos C$.
$BD^2 = (800\tan 40)^2 + (800\tan 60)^2 + 2\cdot 800\tan 40\cdot 800\tan 60 \cdot$ ~~cos 120~~.
$BD = 800\sqrt{\tan^2 40 + \tan^2 60 + \tan 40\tan 60}$

$= 1817$ metres (nearest m) /4

(d)(i) Implicitly differentiate
$2x = 4a\cdot\dfrac{dy}{dx} \Rightarrow \dfrac{dy}{dx} = \dfrac{x}{2a}$

So $m_T = t$; $m_N = -\frac{1}{t}$. Normal is $(y - at^2) = -\frac{1}{t}(x - 2at)$
$ty - at^3 = -x + 2at$
$x + ty = 2at + at^3$

12 (d) (ii)



$X = (2at + at^3, 0)$

$Y = (0, 2a + at^2)$

$TX^2 = (-at^3)^2 + (at^2)^2$

$= a^2t^6 + a^2t^4 = a^2t^4(t^2+1)$

$TY^2 = (2at)^2 + (2a)^2$

$= 4a^2t^2 + 4a^2$

$= 4a^2(t^2+1)$

$\frac{TX^2}{TY^2} = \frac{a^2t^4(t^2+1)}{4a^2(t^2+1)} = \frac{t^4}{4} \Rightarrow \frac{TX}{TY} = \frac{t^2}{2}.$ /2.

End of Q12.

Q13

(a) (i) $\dot{x} = -12\cos t \sin t = -6\sin 2t$

$\ddot{x} = -12\cos 2t.$

$= -12(2\cos^2 t - 1)$

$= -4(6\cos^2 t - 3) = -4(6\cos^2 t - 2 - 1)$

$= -4(x-1)$ as required /2

(ii) $x = 6\cos^2 t - 2 = 3\cos 2t + 1$

$\therefore$ Period $= \frac{2\pi}{2} = \pi$, Centre $= 1$. /2.

13 (b) (i) $a = \frac{dv}{dt} = v\frac{dv}{dx}$ (using chain rule)

So $\frac{dv}{dx} = -\frac{3}{8}x^2$

$\therefore a = -\frac{3}{8}x^2 \cdot -\frac{1}{8}x^3$

$= -3/64\, x^5$ /2

(ii) $\frac{dx}{dt} = -\frac{1}{8}x^3$

$\Rightarrow \frac{dt}{dx} = 8x^{-3}$

$t = 8x^{-2} \cdot (-\frac{1}{2}) + C.$

$t = -4x^{-2} + C.$

At $t = 0$, $x = 2$.

$\Rightarrow C = 1.$

$t = -\frac{4}{x^2} + 1$

$\frac{4}{x^2} = -t + 1.$

$x = \sqrt{\frac{4}{1-t}}$ since $x > 0$, initially. $t \neq 1$, $x \neq 0$. /3

(c) (i) $y = x^2 + 4x - 5$ Inverse defined on either $(-2, \infty)$

$= (x-1)(x+5) \Rightarrow$ intercept of $(1, 0)$ or $(-\infty, -2)$

Interchange 'x' with 'y'

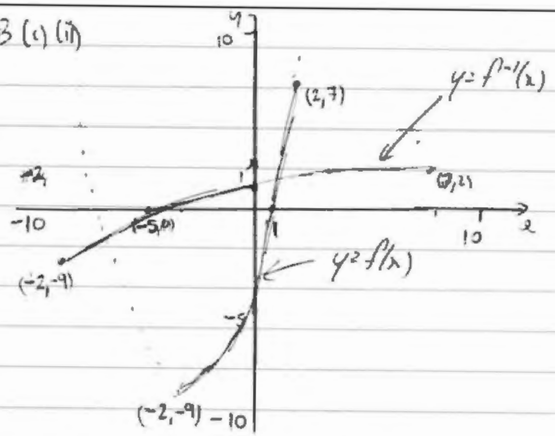
$x = y^2 + 4y - 5$

$= (y+2)^2 - 9.$

$y = \sqrt{x+9} - 2$ (check intercepts of (0,1))

$y = \sqrt{x+9} - 2.$ /1

13 (c) (ii)



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(iii) $y = (x+2)^2 - 9$

$y = \sqrt{x+9} - 2$

$\sqrt{x+9} - 2 = (x+2)^2 - 9$

$\sqrt{x+9} = (x+2)^2 - 7$

$x + 9 = (x+2)^4 - 14(x+2)^2 + 49$

~~$x+9 = \ldots$~~

$x + 9 = (x^2+4x+4)^2 - 14(x^2+4x+4) + 49$

$= x^4 + 16x^2 + 16 + 8x^3 + 32x + 8x^2 - 14x^2 - 56x - 56 + 49$

$x + 9 = x^4 + 8x^3 + 10x^2 - 24x$

$x^4 + 8x^3 + 10x^2 - 25x = 0$

$g(x) = x^3 + 8x^2 + 10x - 25 = 0$, $x \neq 0$.

Divide by $(x+5)$ since $g(-5) = 0$, $x \neq 5$

Gives $x^2 + 3x - 5 = 0$.

$\Rightarrow x = \dfrac{-3+\sqrt{29}}{2}$ (ignore negative since on the graph intersection occurs when $x > 0$.)

End of Q13

/2

Q14

(a) Expand

$a^5x^5 - 5a^4bx^2 + 10a^3b^2x^{-1} - 10a^2b^3x^{-4} + \ldots$

So given coefficient of $x^2 \equiv$ coefficient of x^{-1}

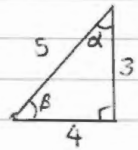
$\Rightarrow -5a^4b = 10a^3b^2$

Divide through by $-5a^3b$

$a = -2b$ or $a + 2b = 0$

/2

(b)



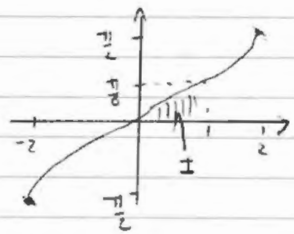
$\tan\beta = \frac{3}{4}$ $\cos\alpha = \frac{3}{5}$

$\beta = \tan^{-1}\left(\frac{3}{4}\right)$ $\alpha = \cos^{-1}\left(\frac{3}{5}\right)$

But $\alpha + \beta = \frac{\pi}{2}$ (Angle sum of triangle)

$\Rightarrow \tan^{-1}\left(\frac{3}{4}\right) + \cos^{-1}\left(\frac{3}{5}\right) = \frac{\pi}{2}$

(c) (i)



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(ii) Integral area shaded above (I)

$I = \frac{\pi}{6} - \int_0^{\frac{\pi}{6}} 2\sin y \, dy$

$= \frac{\pi}{6} + 2\cos y \Big|_0^{\frac{\pi}{6}}$

$= \frac{\pi}{6} + \sqrt{3} - 2$.

/2

14(d)(i)

At $t=2$, $x=8$, $y=-2$ $(t=0, y=0, x=0)$

$2V\cos\theta = 8$ $\quad$ $2V\sin\theta = 19.6-2.$

$V\cos\theta = 4$ $\quad$ $V\sin\theta = 8.8$

$\Rightarrow \tan\theta = 2.2$ $\quad$ /2

(ii) Let $y = f(t)$

$= Vt\sin\theta - 4.9t^2 + 6$ (adjusted ...)

$\dot{y} = V\sin\theta - 9.8t$

But $V\sin\theta = V\cos\theta \cdot \tan\theta$

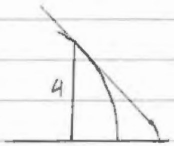
$= \frac{x}{t} \times 2.2$

Using Newtons method approximate $t_0 = 2$.

$t_1 = 2 + \frac{f(t_0)}{f'(t_0)}$

$y|_{t_0} = f(t_0) = 4$

$f'(t_0) = 8.8$



So $t_1 = 2 - \frac{4}{-10.8}$

$= 2.37.$

≈ 2.3 seconds. (since the tangent intersects after the trajectory/projectile hits ground).

End of Q14

Q15

(a)(i) $P(4) = 0 \Rightarrow 64a + 4k - 108 = 0$ (1)

$P(1) = -6 \Rightarrow a + k - 3 = -6 \Rightarrow a + k = -3.$ (2)

From (1) $16a + k = 27.$

$a + k = -3.$

$15a = 30$

$a = 2.$

$k = -5$

So $P(x) = 2x^3 - 7x^2 - 5x + 4$ $\quad$ /2

(ii) $\frac{\alpha\beta + \beta\delta + \delta\alpha}{\alpha\beta\delta} = \frac{+\frac{c}{a}}{-\frac{d}{a}} = -\frac{c}{d}$

$= \frac{5}{4}$ $\quad$ /1

(b) (i) Initial Term $= \log_e a^3 - \log_e b^{1/2}$

$= 3\log_e a - \frac{1}{2}\log_e b$

$T_1 = 3\log_e a - \log_e b$

For arbitrary $n \in \mathbb{N}$.

$T_n = 3\log_e a - \frac{n}{2}\log_e b$

$T_{n+1} = 3\log_e a - \left(\frac{n+1}{2}\right)\log_e b.$

$T_{n+1} - T_n = -\frac{1}{2}\log_e b$

$\therefore$ An arithmetic series, $d = -\frac{1}{2}\log_e b$ $\quad$ /2

(ii) $T_{23} = 3\log_e a - \frac{23}{2}\log_e b$

$T_1 + T_2 + T_3 + \dots = 69\log_e a - \frac{1}{2}(1+2+3\dots+23)\log_e b$

$= 69\log_e a - 138\log_e b$ $\quad$ /2

15 (c)

$u = e^{4x} + 9.$

$\frac{du}{dx} = e^{4x}$ Limits $u_1 = e^{4\ln 2} + 9$

$= 25$

$du = e^{4x} \cdot dx.$ $u_0 = e^0 + 9$

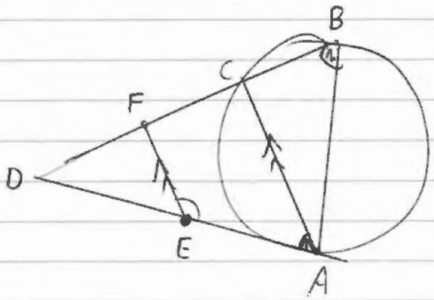
$= 10.$

So $I = \int_{10}^{25} \frac{3}{\sqrt{u}} \, du$

$= \Big[6\sqrt{u} \Big]_{10}^{25}$

$= 30 - 6\sqrt{10}$ /2.

(d) ~~Diagram~~ not,



(i) Angle in alternate segment, standing on the same chord /1

15 (d)(ii) Refer to diagram.

(can use 180° instead of π)

$\angle FEA = \pi - \angle CAE$ (co-interior angles are supplementary where $AC \parallel FE$)

Note $\angle FEA + \angle ABC = \pi$

$\Rightarrow$ EABF is a cyclic quadrilateral since opposite angles are supplementary.

(iii) $\angle BAE = \pi/2$ (or 90°)

(Since AB is diameter, AE is a tangent and angle between tangent & radius is $\pi/2$ (or 90°))

$\Rightarrow$ BE is a diameter (angle in semicircle is $\pi/2$ (or 90°))

End of Q15

Multiple Choice Answers:

Q1.	B	Q3.	C	Q5.	C	Q7.	B	Q9.	B
Q2.	D	Q4.	C	Q6.	D	Q8.	B.	Q10.	D.